

$$-4 \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 1}$$

$$4 \pm \sqrt{16 - 4}$$

$$4 \pm \sqrt{12}$$

$$4 \pm 2\sqrt{3}$$

$$\frac{4 \pm 2\sqrt{3}}{2}$$

$$\frac{4}{2} \pm \frac{2\sqrt{3}}{2}$$

$$2 \pm \sqrt{3}$$

#23

$$|6x^4 - 81|$$

$$(4x^2 + 9)(4x^2 - 9)$$

$$4x^2 + 9 = 0$$

$$\begin{aligned} & \rightarrow -9 \quad -9 \\ & \sqrt{x} \quad \sqrt{x} \\ & \sqrt{4x^2} \quad \sqrt{4x^2} \\ & \sqrt{9} \quad \sqrt{9} \\ & \sqrt{4x^2 + 9} \quad \sqrt{4x^2 + 9} \\ & \sqrt{4x^2 + 9} \quad \sqrt{4x^2 + 9} \\ & \sqrt{4x^2 + 9} \quad \sqrt{4x^2 + 9} \end{aligned}$$

$$(2x + 3)(2x - 3)$$

$$4x^2 - 9 = 0$$

$$4x^2 = 9$$

$$\sqrt{4x^2} = \sqrt{9}$$

$$x = \pm \frac{\sqrt{9}}{\sqrt{4}}$$

$$x = \pm \frac{3}{2}$$

#27 $x^4 + 10x^2 + 9$

$$(x^2 + 1)(x^2 + 9)$$

$$x^2 + 1 = 0 \quad x^2 + 9 = 0$$

$$x^2 = -1 \quad \sqrt{x^2} = \sqrt{-9}$$

$$x = \pm \sqrt{-1}; \quad x = \pm 3i$$

$$x = \pm i$$

$$\frac{P}{Q} = \frac{9}{1} = \frac{\pm 1 \pm 3 \pm 9}{\pm 1}$$

Poss
Rational
zeros $\pm 1, \pm 3, \pm 9$

Calc

No Rational zeros to verify

32

$$f(x) = x^3 + 11x^2 + 39x + 29$$

$$\frac{P}{Q} = \frac{29}{1}; \frac{\pm 1 \quad \pm 29}{\pm 1}$$

Poss. Rat. zeros: $\pm 1, \pm 29$

Calc zeros: $\{-1 - 5 + 2i, -5 - 2i\}$

$$-1 \left| \begin{array}{cccc} 1 & 11 & 39 & 29 \\ & -1 & -10 & -29 \\ \hline 1 & 10 & 29 & 0 \end{array} \right.$$

$$x^2 + 10x + 29$$

$$-10 \pm \sqrt{(10)^2 - 4 \cdot 1 \cdot 29}$$

$$-10 \pm \sqrt{100 - 116}$$

$$x = -5 + 2i$$

$$x = -5 - 2i$$

$$-10 \pm \sqrt{-16}$$

$$\frac{-10 \pm 4i}{2} = \frac{-10}{2} \pm \frac{4i}{2}$$

$$\boxed{-5 \pm 2i}$$

$$(x+1)(x+5-2i)(x+5+2i)$$

$$f(x) = x^4 - 3x^3 + 6x^2 + 2x - 60$$

Given $1+3i$ is a zero find all the zeros.

$$x = 1-3i \quad x = 1+3i$$

$$(x - (1+3i))(x - (1-3i))$$

$$x^2 - x - 3xi - x + 1 + 3i + 3x - 3i - 9i^2 + 9$$

$$x^2 - 2x + 10$$

$$\begin{array}{r}
 x^2 - x - 6 \\
 \hline
 x^2 - 2x + 10 \mid x^4 - 3x^3 + 6x^2 + 2x - 60 \\
 \underline{-x^4 + 2x^3 + 10x^2} \\
 -x^3 - 4x^2 + 2x - 60 \\
 \underline{+x^3 + 2x^2 + 10x} \\
 -6x^2 + 12x - 60 \\
 \underline{+6x^2 + 12x + 60} \\
 0
 \end{array}$$

$$x^2 - x - 6$$

$$(x+2)(x-3)$$

Factor from

$$(x-1-3i)(x-1+3i)(x+2)(x-3)$$

zeros $1-3i, 1+3i, -2, 3$